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Prediction of low-cycle fatigue behaviour of GFRP: an experimental approach

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Fatigue tests conducted on "Scotchply-1000" under constant stress range ΔS or constant strain range $\Delta\epsilon$ gave essentially the same life when the stress range was normalized by the ultimate strength ($\Delta S/\sigma_u$) and the strain range was normalized by the fracture strain ($\Delta\epsilon/\epsilon_f$). Fatigue cycling of either type produces a progressive decrease in the modulus of elasticity which is a linear function of log N. An empirical relation of the type used to predict the low-cycle fatigue of metals was selected to predict the low-cycle fatigue behaviour of GFRP materials. Agreement between the experimental results and the predictions of the empirical relation was found to be good. A general method of evaluating the constants from a limited number of fatigue tests has been suggested. Further generalization of the constants and their correlation with the tensile properties of the material may be possible with the availability of more data on the fatigue of composite materials.

1. Introduction

Glass reinforced plastics are being utilized in increasing proportions due to a number of favourable material characteristics. These include high specific strength, resistance to corrosion, electrical and thermal insulation and formability. The high specific strength of composites with controlled directional reinforcement can be favourably compared to metals commonly employed in structural elements. However, in some cases the structures are subjected to a finite number of load or displacement cycles during its useful life and the property governing the efficient use of a material for these applications is its low cycle fatigue strength.

As the utilization of composite materials in structural applications is relatively new, their behaviour in low-cycle fatigue is not well understood. Experimental data necessary to develop standard design procedures are still lacking. The objective of this study was to gather fatigue data for a specific GFRP material in the low-cycle range and to evolve an empirical relation which could predict the low-cycle fatigue behaviour.

Studies by Manson [1] and Coffin [2] have clearly indicated that the fatigue life of metallic materials is controlled by the strain range in the low cycle region. More recent investigations by James *et al.* [3] with composite materials are not conclusive regarding the dependence of cyclic life on either the stress or the strain.

The work by Broutman, McGarry and associates [4-7] has illustrated many features of the failure mechanism in GFRP materials. In all cases the matrix of the fibrous composite exhibits widespread microcracking prior to gross failure under both static and cyclic loading. The microcracking appears to be due to the low toughness and low strength of the resin coupled with the severe strain concentrations produced by the glass fibres embedded in the matrix. More recently, Broutman and Sahu [8] determined that damage in a cross-ply composite initiates in the cross-ply and under the action of cyclic loads further cracking develops in the longitudinal-plies prior to delamination and subsequent failure. Owen *et al.* [9, 10] have shown that crack initiation in chopped strand laminates occurs by debonding within strands of fibres located perpendicular to the line of load. Finally, Dally and Carrillo [11] studied the fatigue behaviour of four glass fibre fortified thermoplastics with different fibre lengths and matrix materials. The fatigue mechanisms associated with the four composites were markedly different. Fatigue damage initiated in all four materials by a debonding mechanism but the propagation of fatigue cracks was controlled by the fibre distribution, orientation and the toughness of the matrix materials.

The present experiments were designed to give sufficient data to establish the dependence of low cyclic fatigue life on either the stress or the strain range. To avoid the effect of hysteresis heating, the fatigue testing was performed at low frequencies on a servo-controlled hydraulically actuated testing system. The stress and strain ranges and the number of cycles to failure were measured in all tests. However, in one series, the machine was cycled between the constant stress limits and the strain range was recorded. In the second series, the strain range was controlled, and the stress range was recorded.

2. Experimental procedure

2.1. Materials

The material selected for the study was an E-glass fibre reinforced epoxy, commercially known as "Scotchply-1000" and marketed by the "3-M Company" of USA. The material was prepared in plate form with an $\frac{1}{8}$ in. thickness from 15 plies of a preimpregnated unidirectional web. The panels were prepared with a cross-ply glass orientation having seven longitudinal-plies and eight cross-plies. The panels were cured in a platen press at 100 psi* for 35 min at 330°F (165°C) and then postcured in an oven at 225°F (108°C) for 16 h. The cured plates exhibited a resin content of 29.6% by weight in a burn-out test.

The panels were cut with a water-cooled diamond plated wheel into rectangular blanks $\frac{3}{4} \times 8$ in. in size. These blanks were then machined to the shape of an ASTM tension specimen (ASTM No. D638-61T) by using a template and a diamond-plated rotary file on a routing machine.

The static behaviour of the material was established by testing five specimens on an Instron testing machine, where the stress-strain curves were directly recorded. Typical results are shown in Fig. 1, where it is noted that the stress strain curve is non-linear. However, the curve can be approximated by two straight lines which intersect in the neighbourhood of $\sigma = 36\,000$ psi and $\epsilon = 1.0\%$. The initial modulus, E_1 was 3.6×10^6 psi and the final modulus, E_2, was 2.7×10^6 psi. The average ultimate tensile strength of the material was 66 000 psi and the average ultimate strain was 2.1%.

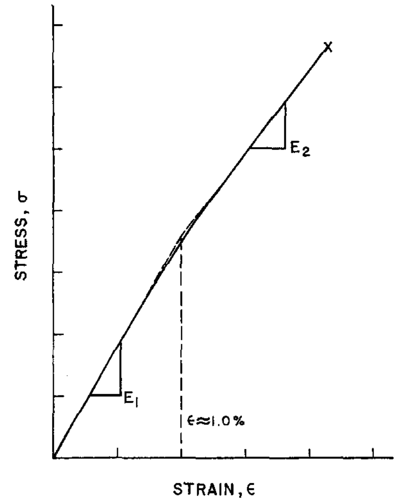

Figure 1 Stress-strain curve for E-glass epoxy cross-ply laminate.

2.2. Fatigue testing procedure

The fatigue tests were designed to give the data necessary for establishing the dependence of low cyclic fatigue life on either cyclic stress or cyclic strain. These data were obtained by performing two series of fatigue tests, one by cycling specimens between stress limits and recording the strain data, and the other by controlling the strain range during the cyclic loading and recording the resulting stress range. All the tests were performed at sufficiently low frequencies to avoid excessive hysteresis heating of the specimen.

The low-cycle fatigue tests were conducted on a servo-controlled hydraulically actuated testing system which was operated between frequencies of 0.01 and 2 cycles per second (Hz). The stress controlled tests were carried out in fluctuating tension with the minimum stress equal to 5% of the maximum stress ($R = 0.05$). During the stress controlled tests, the strains were recorded at periodic intervals with a 2 in. extensiometer.

The specimens, used to establish the high cyclic fatigue life, were tested on a Budd Double-Direct fatigue testing machine. This machine was operated at frequencies between 300 and 600 cycles per minute. The load is maintained on the

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*10^3 psi = 6·89 N mm^{-2}

specimen during test by a hydraulic loading system controlled by limit switches contacted by the loading beam.

A total of 65 specimens, 45 in the low cycle range (up to the cyclic life of 10^4) and 20 in the high cycle range (cyclic life of 10^4 to 10^6), were tested. The S–N curve defined over a range of N from 3 to 10^6 cycles is shown in Fig. 2.

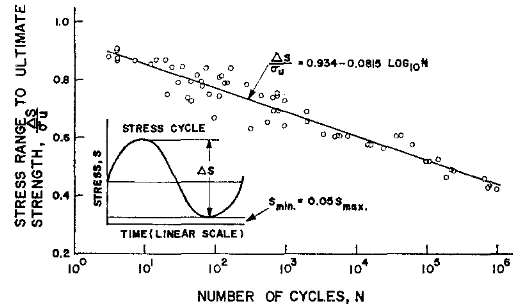

Figure 2 Fatigue strength of GFRP under stress controlled tests ($R = 0.05$).

The strain controlled tests were also performed on the servo-controlled machine. The minimum strain was maintained at 5% of the maximum strain. The strain range was controlled with a cantilever type displacement transducer attached to the hydraulic actuator. However, the strain range was monitored and manually adjusted by using an extensiometer and recorded at periodic intervals. Forty-five specimens were tested to establish the $\Delta\epsilon$–N curve ($7 < N < 10^4$) which is shown in Fig. 3.

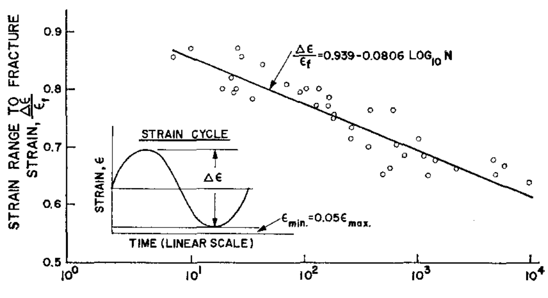

Figure 3 Fatigue strength of GFRP under strain controlled tests ($R = 0.05$).

3. Results and discussion

3.1. Cyclic stress

The ΔS–N curve for this material shown in Fig. 2 is similar to the classical S–N curve obtained for metals [12], polymers [13] and composite materials [8]. These results indicate that the fatigue strength as indicated by the ratio of the stress range ΔS to the ultimate strength σ_u is a linear function of the log of the cyclic life given as:

$$\frac{\Delta S}{\sigma_u} = 0.934 - 0.0815 \log_{10} N. \qquad (1)$$

This relationship was established by a regression analysis of the data and showed a coefficient of correlation of 0.952.

The fatigue strength, in terms of stress range ΔS, at $N = 10^6$ is about 0.445 σ_u with no indication of an apparent endurance limit. As indicated by the regression equation, the rate at which the strength is decreasing is essentially constant over the range of cyclic life considered here at $d(\Delta S/\sigma_u)/d \log_{10} N = 0.0815$.

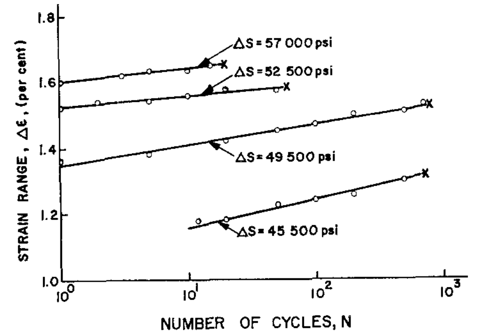

Figure 4 Strain-range as a function of the number of cycles of fatigue exposure at constant ΔS.

The variation in strain range with the number of cycles for the specimens cycled between fixed stress limits is shown in Fig. 4. The results are shown for four stress ranges, namely ΔS = 45 500, 49 500, 52 500 and 57 000 psi. All the results indicate that the strain range increases for a constant stress range as the number of cycles of loading is increased. The variation of modulus ($E_\sigma = \Delta S/\Delta\epsilon$) with the number of cycles has been obtained for the above stress ranges as shown in Fig. 5. The fatigue modulus decreases with an increase in the number of cycles in all cases. This behaviour is similar to strain-softening noted in metallic materials; however, the mechanism for producing this softening is markedly different from that in the metallic materials. Moreover, in case of metallic materials modulus tends to stabilize before half the number of cycles to failure has been applied. In the case

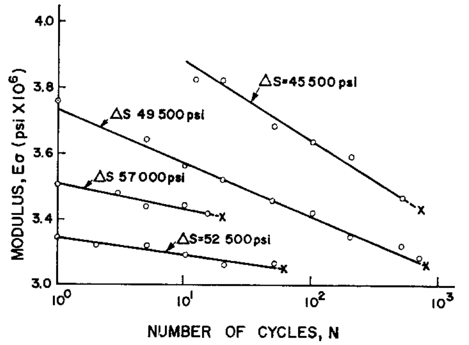

Figure 5 Fatigue modulus as a function of the number of cycles of fatigue exposure at constant ΔS.

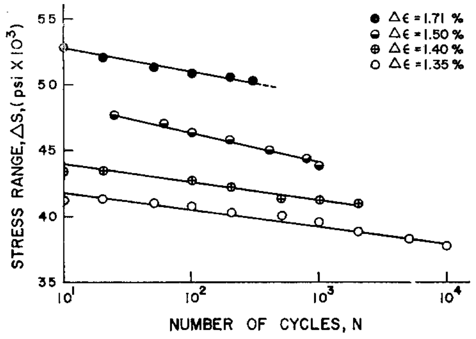

Figure 6 Stress range as a function of the number of cycles of fatigue exposure at constant $\Delta\epsilon$.

of composite materials no such stabilization has been observed. The modulus in composite materials continuously decreases as the fatigue exposure continues until failure occurs. This continuous degradation of the fatigue modulus is known to be due to the progressive cracking of the cross-plies [14].

3.2. Cyclic strain

The results obtained for the GFRP material under strain controlled tests are shown in Fig. 3 in the form of a curve of $\Delta\epsilon$ versus N. Again it was found that the loss of fatigue strength expressed as the ratio of the strain range $\Delta\epsilon$ to the ultimate strain at fracture, ϵ_f, was a linear function of the log of cyclic life

$$\frac{\Delta\epsilon}{\epsilon_f} = 0.939 - 0.0806 \log_{10}N. \qquad (2)$$

This equation showed a coefficient of correlation 0.90 in a regression analysis.

The strain range required for failure at $N = 10^4$ cycles is about 0.617 ϵ_f and it is decreasing rapidly for an increased number of cycles. Like the results for stress cycling, the rate of decreasing strain capacity $d(\Delta\epsilon/\epsilon_f)/d(\log_{10}N)$ appears to be constant at 0.0806 over the entire range of cyclic life considered here.

The variation in the stress range with the number of cycles at a fixed strain range is shown in Fig. 6. The results are presented for four strain ranges namely $\Delta\epsilon$ = 1.35, 1.4, 1.5 and 1.71 %. All of the results show a decrease in the stress range with an increase in the number of cycles. In this case also, the variation in the fatigue modulus ($E = \Delta S/\Delta\epsilon$) with the number of cycles has been determined as shown in Fig. 7.

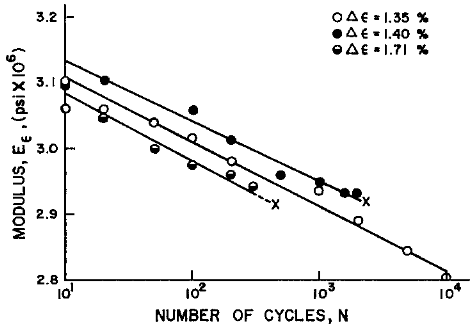

Figure 7 Fatigue modulus as a function of number of cycles of fatigue exposure at constant $\Delta\epsilon$.

It is again noted that the fatigue modulus decreases continuously with the number of cycles of fatigue exposure.

The ΔS–N and the $\Delta\epsilon$–N relations, which have been normalized by the ultimate strength, σ_u, and the fracture strain, ϵ_f, respectively, are compared in Fig. 8. Considering the large scatter in the fatigue data, these two lines are essentially the same. Thus, it appears that the stress range or the strain range are equally important in the low cycle fatigue behaviour of GFRP materials. These results indicate that tests conducted under stress control can be used to predict results of tests under strain cycling and vice versa.

This observation is different from that for metallic materials, where low cycle fatigue is entirely controlled by strain range. This behaviour of GFRP materials can be explained on the basis of the elastic response over the strain range considered here. Either the stress or strain cycling damages the composite producing a decrease in the modulus of elasticity. However, in spite of this damage the response of the material remains

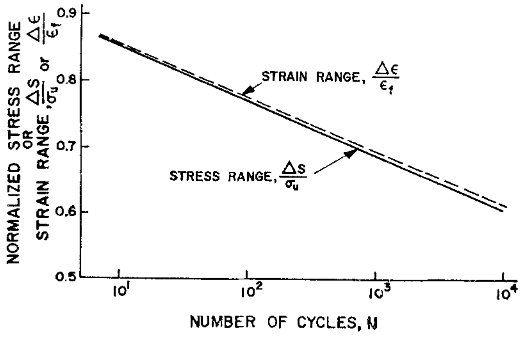

Figure 8 Comparison of low-cycle fatigue strength for stress and strain controlled tests.

essentially elastic, and the effect of ΔS or $\Delta\epsilon$ can be equated by the normalizing with σ_u and ϵ_f.

3.3. Comparison of results with empirical relation

The fatigue behaviour of metals is relatively well understood, and the mechanism involved for initiation of a fatigue crack is generally agreed upon. Studies by several investigators have clearly indicated that the fatigue life of metallic materials is controlled by the strain range in the low cycle region. It has also been established that plastic strain is a more sensitive measurement of life than the nominal stress. This is because of the fact that the yield strength of the metallic materials is lower than the stresses required to produce failure in low cycle fatigue. The stresses associated with the high cycle range are usually lower than the yield strength and the stress-strain relationship is linear. Therefore, stress controlled and strain controlled tests give essentially the same results in the high cycle range.

There have been several studies to predict the S–N or $\Delta\epsilon$–N curves of metallic and polymeric materials either from a simple tension test or from a limited number of fatigue tests. Investigations of Manson [1] and Coffin [2] are well known. Both investigators have proposed relationships between plastic strain range and cyclic life. The experiments on a large number of metallic materials indicate that the predictions are reasonably good.

Coffin [2] has proposed the following equation to predict the strain cycling data:

$$N^k \Delta\epsilon_p = c \tag{3}$$

where N = cycles to failure, $k = \frac{1}{2}$, $\Delta\epsilon_p$ = plastic strain range. The parameter c can be related to the reduction in area as follows:

$$c = -\tfrac{1}{2} \ln \frac{100 - \text{R.A.}}{100} \tag{4}$$

where R.A. is the per cent reduction in area.

Tavernelli and Coffin [15] have evaluated c for twelve metallic materials which were investigated in low cycle fatigue by various investigators. These values of c range from 0.098 for AISI 422 stainless steel to 0.855 for 2S aluminium annealed. Tavernelli and Coffin further suggested that to obtain total strain range, the elastic portion should be taken as $2Se/E$ where Se is the endurance limit and E is the modulus of elasticity.

Manson [1] has expressed the strain range in terms of the elastic and plastic components and has shown that both of them are linear when plotted against cyclic life on log-log co-ordinates. Two methods have been suggested to obtain these two straight lines. In the first method, the two straight lines are obtained by a procedure utilizing properties related to the tensile behaviour of the material. An alternative approach to obtain the two lines is more readily adoptable to the present case. In this approach, the straight lines obtained by plotting the elastic strain range and plastic strain range against the cyclic life, are assumed to have the same slopes for all materials. These relations proposed by Manson can also be expressed in the form similar to that given by Coffin as:

$$N^{k_1} \Delta\epsilon_e = c_1 \tag{5}$$

$$N^{k_2} \Delta\epsilon_p = c_2 \tag{6}$$

where $k_1 = 0.12$, $k_2 = 0.6$, $\Delta\epsilon_e$ = elastic strain range, $\Delta\epsilon_p$ = plastic strain range, $c_1 = 3.5\ \sigma_u/E$, $c_2 = (2c)^{0.6}$.

Owing to the absence of plastic strain in the composite material, the total strain range and the cyclic life may be related directly by a law similar to the ones proposed by Coffin and Manson:

$$N^{k_3} \Delta\epsilon = c_3 \tag{7}$$

where $\Delta\epsilon$ is the total strain range and k_3 and c_3 are material constants which can be evaluated by fatigue tests. The data for the composite material were compared with the predictions of Equation 7 in Fig. 9 and constants k_3 and c_3 calculated by satisfying Equation 7 at $N = 100$ and 1000 cycles. The constant k_3 was found to be 0.0475 against 0.5 in Coffin's equation, 0.12 in Manson's equation for elastic strain range and 0.6 for plastic strain range. The constant c_3 was found to be 0.0203 against the values ranging

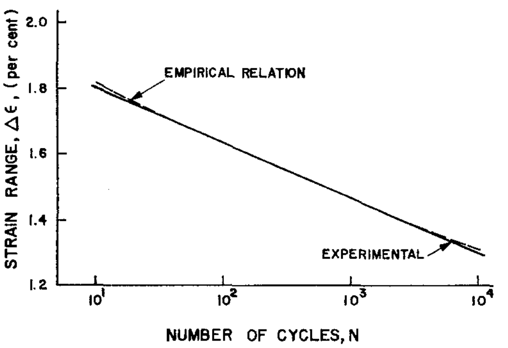

Figure 9 Comparison of experimental data with the empirical relation.

from 0.098 to 0.855 given by Tavernelli and Coffin for metallic materials. The constant c_1 of Manson's equation for elastic strain range was found to be 0.064 for the composite material. To calculate constant c_2 of Manson's equation for plastic strain range, the ductility, D, was obtained from the knowledge of constant c of Coffin's equation. The constant c_2 was found to be ranging from 0.376 to 1.38 for metallic materials. It is apparent that the values of the constants k_3 and c_3 for composite materials can be compared with those for metallic materials. Differences can be attributed to the fact that the composite material under present investigations is completely different from metallic materials. A close examination of the above numbers shows that the values of constants k_3 and c_3 obtained for the present data are closer to the ones for Manson's equation for elastic strains (Equation 5) than either for Manson's equation for plastic strains (Equation 6) or for Coffin's equation (Equation 3). In view of the elastic behaviour of the composite materials, this is an expected result.

It may be speculated that the constant k_3 be a universal constant for all glass fibre reinforced plastics and that the constant c_3 be related to the material properties measured in a tensile test as is the case for metallic materials. However, decisive conclusions regarding the constants k_3 and c_3 and their possible correlation with the tensile properties can be drawn only after a considerable amount of fatigue data are produced for composite materials. The present studies are a step to that end.

4. Conclusions

Fatigue tests conducted under constant stress range, ΔS, and constant strain range, $\Delta\epsilon$, produced essentially the same results when ratios $\Delta S/\sigma_u$ and $\Delta\epsilon/\epsilon_f$ were used to normalize the data. These results strongly suggest that either strain controlled or stress controlled fatigue tests can be used in low-cycle fatigues studies of GFRP. In all cases, a progressive decrease in the fatigue modulus was observed. It was noted that this decrease was essentially a linear function of log N.

An empirical relation of the type used to predict the low-cycle fatigue behaviour of metals was selected for GFRP materials also. Experimental results for the particular GFRP compared favourably with the predictions of this empirical relation. A generalization of the empirical relation and establishment of a correlation between the constants of the equation and the material properties may be possible in future when a large number of data are available on the fatigue of composite materials.

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